including a shear strength correction to the single-crystal shock data. This correction to the single-crystal data was based on shear strengths measured in shock compression experiments on polycrystalline Al_2O_3 and interpretations based on the elastic-plastic model of the deformation of solids. Isothermal, isotropic compression data derived from the single-crystal shock data were compared with extrapolations of polycrystalline ultrasonic data. A recent study of the shock compression of sapphire [*Graham and Brooks,* 1971] obtained new high-pressure data between 175 and 420 kb and also evaluated the shear strength of sapphire without assuming that the elastic-plastic model is valid.

It is the purpose of this article to utilize the recent shock data between 175 and 420 kb [*Graham and Brooks,* 1971], the ultrasonic data [*Gieske and Barsch,* 1968], and the very-high-pressure shock data between 500 and 1500 kb (R. G. McQueen and S. P. Marsh as reported by *Anderson* [1966]) to test the linear bulk modulus approximation for sapphire.

Compression Equations

If it is assumed that the isothermal bulk modulus is linear in pressure, the expression for compression can be obtained by integrating (1) to obtain the Murnaghan equation,

$$P = \frac{B_0}{B_0'}\left[\left(\frac{V_0}{V}\right)^{B_0'} - 1\right] \qquad (2)$$

where V is the specific volume at pressure P and V_0 is the specific volume at atmospheric pressure. Thus, to the extent that the linear bulk modulus approximation is valid, the results of ultrasonic measurements can be used in (2) to construct an isothermal equation of state to very high pressure.

Even though (2) describes compressions over a wide pressure range, it is computationally more convenient to use polynominal expansions of relative volume about the initial volume to obtain bulk modulus values from compression measurements. For moderate pressures, the salient features of compression can be described by a cubic polynomial expansion,

$$\frac{V}{V_0} = 1 - A_1P + A_2P^2 - A_3P^3 \qquad (3)$$

Anderson [1966] pointed out that in the linear bulk modulus approximation the coefficients of (3) are not arbitrary constants to be empirically determined, but should be expressed in terms of the bulk modulus and its pressure derivative. In particular,

$$A_1 = 1/B_0 \qquad A_2 = \tfrac{1}{2}(1 + B_0')/B_0^2$$

$$A_3 = \tfrac{1}{6}(1 + 3B_0' + 2(B_0')^2)/B_0^3$$

Graham and Brooks [1971] were able to fit their shock data with (3) to obtain shock compression values for B_0'. This value of B_0', which is independent of that obtained ultrasonically, can be confirmed by comparison to the very-high-pressure shock measurement of McQueen and Marsh. However, before proceeding with this comparison, the shear strength effects must be considered. Only a brief summary of these considerations will be presented; more extensive discussions can be found in *Jones and Graham* [1971] and *Graham and Brooks* [1971].

Shear Strength Effects

If the shear strength of a solid is nonzero, the state of stress achieved in the plane-wave shock experiment will be anisotropic. Accordingly, the high-pressure shock experiment will show a stress or volume offset from the isotropic compression curve. Many solids exhibit offsets that are proportional to the yield strength of the solid. Yield strengths of shock-loaded solids, called Hugoniot elastic limits, are now routinely measured, and a recent summary of these measurements [*Graham and Jones,* 1968] showed many solids with typical values from a few kilobars to several tens of kilobars.

Hugoniot elastic limit values for sapphire depend on the crystallographic orientations of the samples; values as large as 210 kb have been observed [*Brooks and Graham,* 1966]. Thus, shear strength offsets potentially as large as 3% in volume at a given stress are possible above the Hugoniot elastic limit. However, the recent shock compression study of sapphire [*Graham and Brooks,* 1971] examined these shear strength effects in the region above the Hugoniot elastic limit and found that sapphire exhibits a substantially lower shear strength in the high-pressure region than predicted from the Hugoniot elastic limit measurements. These shock compression data show that a relative volume correction, η_0, of $0.0118V_0$, corresponding to a stress offset, σ_τ, of 30 kb, must be applied to the shock data up to 420 kb to correct for the anisotropic component of the compression. Furthermore, various crystallographic orientations showed a common high-pressure compression curve. This latter observation is particularly important since the data of McQueen and Marsh were obtained on samples with unknown crystallographic orientations.

Although this shear strength correction is characteristic of pressures up to 420 kb, the correction cannot be extrapolated to the shock data at pressures as high as 1500 kb unless the pressure dependence of the shear strength is known. Various forms of the pressure dependence of the shear strength can be assumed; a constant shear strength or an increasing shear strength is usually assumed, but a decrease in shear strength cannot be unequivocally ruled out. Even though there is no quantitative guide for selecting the most appropriate shear strength model, shear strengths are generally thought to increase with increasing pressure. Thus, the constant volume offset assumption will be used to correct the shock data since it has the effect of applying an increasing shear strength correction with increasing pressure. Other shear strength models are not precluded; in fact, the various models serve to emphasize the uncertainties involved in evaluating shear strength effects in shock-loaded solids.

COMPARISON OF SHOCK AND ULTRASONIC DATA

Considerable information is now available to test the linear bulk modulus approximation for sapphire. The bulk modulus and its pressure derivative have been accurately determined ultrasonically and two independent shock-compression investigations provide data from 175 to 1500 kb. Furthermore, the effects of shear strength have been evaluated, and, since the thermal pressure correction due to shock heating is small, uncertainties in the details of the equation of state are not significant.

Various isothermal compression parameters for single crystal Al_2O_3 are shown in Table 1. Values for high-density polycrystalline Al_2O_3 are shown for comparison. The linear bulk modulus approximation can be tested in several independent ways. For example, the ultrasonic values from Table 1 can be used in (2) and the calculated compression curve compared with the isotropic, isothermal compression curve calculated from shock compression data. On the other hand, the modulus values determined from the low-pressure shock work can be used in (2) and the calculated compression curve compared with the very-high-pressure shock results. It should be noted that the shear strength values obtained on single-crystal Al_2O_3, as shown in Table 1, are less than the polycrystalline values previously observed.

The shock data are corrected to isothermal conditions by calculating the thermal pressure resulting from the shock compression. Fortunately, the thermal pressure is small for sapphire, amounting to only 40 kb at a shock pressure of 1000 kb. Since the thermal pressure is small, errors in thermal pressure calculation are not significant and uncertain details of the equation of state cause uncertainties in thermal

TABLE 1. Isothermal Compression Parameters for Al_2O_3

			Modulus Values		Shear Strength Values	
Source	Technique	Material	B_0, kb	B_0'	σ_τ, kb	η_0
Schreiber and Anderson [1966]	Ultrasonic	Polycrystal	2504	4.00		
Gieske and Barsch [1968]	Ultrasonic	Crystal	2526	4.35		
Ahrens et al. [1969]	Shock	Various	2901	3.24	52	
Graham and Brooks [1971]	Shock	Crystal	(2526)*	3.72	30	0.0118

Conversion of adiabatic to isothermal parameters was accomplished with the thermodynamic values reported by *Anderson* [1966]. B_0 is the isothermal bulk modulus, B_0' is the pressure derivative of the isothermal bulk modulus, σ_τ is the shear stress offset, and η_0 is the relative volume offset due to the shear strength effects.

* Independent value was not obtained in the shock experiments. The fit to the shock data was assumed to have a value equal to the ultrasonic value.